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Infinity, limits and order: a study on the paradox

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INFINITY, LIMITS AND ORDER

A STUDY ON THE PARADOX

ΑΠΕΙΡΟΝ, ΟΡΙΑ ΚΑΙ ΤΑΞΙΣ

ΜΙΑ ΜΕΛΕΤΗ ΓΙΑ ΤΟ ΠΑΡΑΔΟΞΟ

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INFINITY, LIMITS AND ORDER

A STUDY ON THE PARADOX

In a paper which deals with the paradox of infinity the most fundamental paradox is the paper itself. For it deals with the infinite in a finite time and extension. It tries to limit the unlimited, to define the indefinite, probably the undefinable. This basic preliminary remark is not meant to be a pleasant beginning, nor is it meant to be an easy excuse. It is a basic beginning which structures our paper and our aims in it.

The paradox of infinity (as the paradox of freedom, or every good paradox) is such that it challenges the whole enterprise of knowledge and discourse on all possible levels, abstract or concrete, particular, universal or individual. It challenges all attempts at interpretation or formulation (even of the problem of infinity) as long as they rest on limits and order. It is relevant to each step of our argument, or in the arguments of the thinkers that we present in this paper. Consequently it is impossible to maintain any illusion for a completeness, since not only passages which deal with infinity itself, but all passages, all structured thoughts as long as they define and order, (as long as they exist merely) are relevant for consideration of the paradox. Even if we took into consideration the complete works of an author we would never be able to account for a complete analysis of all parts, and furthermore there could be an indefinite number of interpretations that we have not examined. The word completeness is a limit-word running sometimes contrary and sometimes parallel to the notion of infinity. In this paper we want to trace relations of meaning and inference between infinity and knowledge and since we are far from completeness, we are only setting up beginnings. We need to set up ways of dealing with this paradox and relate these to the enterprise of knowledge.

This paper is a beginning, a setting up of possibilities of directions (these directions may progress ad infinitum or they may vary indefinitely). A more complete study of infinity will be triggered by this beginning paper and will be based, on the one hand, on the beginning made in this paper (beginning is a limit and consequently it will be a further consideration of the thinkers presented in the semantics part and a further analysis of the

problems, which is comparable to infinite by division), and on the other hand it will be based on the general conclusions reached about the relations of infinity to knowledge (conclusions is a limit word and consequently in this sense the paper will progress towards infinity by addition).

To return to the fundamental paradox of the paper. This paper is a form, is limited, it has beginning and end and magnitude, it is also ordered. But the subject it treats may be infinite. The paper may be analyzed ad infinitum or completed ad infinitum and it may be the case that we can go further or deeper (narrower) because the subject is infinite. Or even, it may be the case, that we can go further and deeper because the subject it treats (the «out of which») is finite but there can be infinite numbers of treatments of it. In any case, this extension of the paradox to the paper itself is not insignificant, for it may be that after such beginning the conclusions will be in accordance.

To make it clearer, given the fundamental paradox, what are we to do? We can stop here so that we will not be in controversy. But semantically speaking, even though philosophers were aware of the paradox, they continued their search for a definition of infinity. It must then be the case that there is something in the nature of the paradox, as such which relates to knowledge basically, even though the paradox tries to undercut the relations of infinity and possibility of knowledge. Actively, then, we enter the discussion. Given the paradox of infinity, our aim now is to search for the relations of the paradox to knowledge (specific or general). Do people avoid the paradox or do they insist on it? Is the paradox the limit of our knowledge or is it the point at which we step out of the limits? Is it a boundary from the inside or from the outside, is it a boundary at all? Is it a beginning or an end of knowledge? Is it related to knowledge at all? What are the consequences of it? We break the original deadlock by starting the paper, knowing that to start means to make a beginning, to set a limit.

Having made our beginning, even if paradoxically, we can talk about infinity now. We do not forget the fundamental paradox, we will see how our treatment of infinity affects the fundamental paradox at the end. In the following pages we will look at the paradox of infinity as a subject matter to our paper. We have delimited this area by a finite number of problems we have asked and will ask about infinity, by a finite number of thinkers and writings, by a finite number of interpretations of these thinkers, by setting up proportions of finite ratios of terms (ὅρος), by analysing down to finite simples, and by synthesizing our whole, the paper out of finites (simples, words, wholes, problems, thoughts).

If we make a very broad presupposition, that human understanding operates by limiting (defining) and ordering, we can say that the paradox of infinity lies in our attempt to define the indefinable, which is to limit the unlimited; to order the infinite will be like creating an infinite regression or a division down to the indivisible. In this way, the infinite presents a challenge to the whole activity of understanding. But it is not only an abstract epistemological problem that we are faced with. The range of the paradox is very wide. Because in all the range of knowledge as long as people make a beginning, follow a sequence, and end up in a conclusion there is bound to be a problem of infinity.

The paradoxes of motion as stated by Zeno are a good illustration of the paradox of infinity when applied. Furthermore, the paradox of the «now», or the motion in the present, or the change from rest to motion, or from motion to rest. The paradox of determinism and freedom as stated by Tolstoi in his second epilogue to *War and Peace* is also an instance of the application of the paradox of infinity to history. It is extended to the problem of how can we know all the determinants of a situation, from where we can point to the problem of complete or incomplete induction as a more general statement of the problem, and from there more paradoxes are in line, such as the indeterminacy principle of Heisenberg. The range of the problem runs in a two-directional manner from considerations of the continuous to considerations of the one (unity) and extends over microcosm and macrocosm. The most common example is the question-paradox of the infinity of the universe: where does it end? what is beyond? and also the question-paradox of the quanta physics, about the finite division and the indivisible.

We made a beginning by saying that the human mind understands by limiting, by defining. A form is taken as something limiting or limited. If the paradox exists in motions and actions which we experience, is it possible that it would not exist in a discipline which tries to keep only to the formal and away from enmattered forms and experience? Is it possible that we took the formal infinity out of a formal science (or a science of forms and actuality) and applied it to experience, creating thus a paradox out of nothing? For it may be argued that it is the case that because of the infinity of numbers or the continuity of lines (which are by definitions formally set) and because of the analogies that we make to the world that paradoxes arise in the consideration of motion, time etc. The case may be argued in an opposite way toot that from the infinite in the world we move to the infinity of numbers bu, for our purposes it is significant to realize that both directions point to the analogy as the seat of the paradox. That such positions exist it is enough to cite the example of the Euclidean Space-Euclidean Universe analogy, which

of course cannot be based on the writings of Euclid. There may be a significance in the relations of world to mathematics but let us, for the time being, examine the notion of infinity in mathematics, as a formal science. It can be argued that any notion in mathematics is to be understood and used as long as it is defined. We can define (form) an infinity or an infinite group, and, as long as we do not make analogies to experience, we will face no paradox. This is not though, exactly the case in mathematics, and there the notion of infinity is not a non - puzzling form of actuality. Euclid, to refer to our example, finds in infinity the limits of his constructed system. He can construct finite structures by line segments and angles but the limits of constructibility (Q.E.F.) is infinity (parallel lines), while on the other end of the line the ambiguous indivisible point is a simple and a finite. Infinity appears therefore as the end of formality and constructed actuality, and is not included in it as a form. At these limits of finitude (that is, infinity) a lot of paradoxes happen. If we extend the radius of a cycle ad infinitum (to an actual infinity) then we will have a circumference which is a straight line. If we divide the circumference of a cycle ad infinitum we will reach a point where the infinitesimal line segments are straight line segments. The parallel lines meet at infinity while the π and the irrational numbers have always been a source of puzzlement. In all the cases we are faced with these paradoxes because we have taken the infinite as an actuality, as a defined form.

In arithmetic and algebra the infinite is again a source of paradoxical situations. For it cannot enter into measures or into analogies and proportions. An a over ∞ and a b over ∞ are equal even if $a \neq b$. The infinite cannot become a term in an analogy a ratio or a proportion because a term (ὅρος) even if it is variable is something limited. The situation ∞/∞ is called indeterminacy. The problem is solved partly by the theory of limits and the indeterminacy is removed by replacing ∞ by $\rightarrow \infty$ which is called «tends towards infinity», or «has infinity as a limit». But then the problem still remains and the infinite is not an actuality. Another example is furnished by Galilei where the series 1, 2, 3, 4,... is infinite and as such is equal to the series 1, 4, 9, 16,... which is also infinite and the paradox is then that the part is equal to the whole. But the infinite does not admit of parts and wholes, since parts and wholes are in need of limits, in need of forms. The same paradoxical situations are reached when we take the $\delta t \rightarrow 0$ where the limit is the infinite divisibility.

It seems then, that if we want to remain in the mathematical infinite as an actuality, we will have to talk of a new mathematics system (probably of arbitrariness), and create a new science of the actual infinity. Cantor and Hilbert have worked on such a programm and the «finite - infinite - trans-

finite» give supposedly an idea of infinity which is not negative nor potential. But, is it arbitrary? The criticism of Poincaré seems to point to this direction. If arbitrariness and formality can be properly put together in a mathematical system or if there are even other paradoxes in such a system, I leave to the people skilled and inquisitive about mathematics. For our purposes mathematics, even if formal, does not further this discussion, except by arbitrariness. The only point which is of interest to us are the criteria of arbitrariness in general as related to forms and definitions delimitations and limits (which will be studied later in the paper).

In another area of arbitrariness (the dogmatism of faith), the infinity of God provides paradoxical situations comparable to the previous ones. The infinite power, knowledge, and goodness of the divine comes to contrast with the experienced reality which is seen as finite. Out of this situation we can escape either by the arbitrary claim that the ways of the divine are not to be understood by finite minds or, if we want to inquire further (which is not dogmatic faith but theology), we may end up with principles that lead to various theories such as pantheism, or any other product of sophisticated theology which leads up all the way to metaphysics. In this case faith is of a different quality and the paradox of infinity is not rejected by faith. In this paper we will keep away from arbitrary actualities, and thus we will be limited to Physics (Natural Philosophy), Epistemology and Metaphysics.

The range of the paradox is not extended to different areas of knowledge only, but is extended and modified in the different epochs. For the ancients infinity is viewed as imperfection. The lack of actualized form (as in the things) in the infinite is the basis for such a modification of the infinite. In the poem of Parmenides Being is finite because it is perfect, and consequently does not stand in need of limits, while the infinite, being always in need, is imperfect, and thus it is not a predicate of Being. Of course, the imperfection of the infinite is not a general rule for the antiquity (Melissos considers Being as infinite), nor is this tendency limited to antiquity alone. The criticism of Einstein to the indeterminacy principle and the complementarity principle of Heisenberg and Bohr respectively can be traced to this aspect of the paradox of infinity. The reverse is the case after the influence of christianity and the monotheistic religions. Infinity is joined with perfection as an attribute of the divine. It is only the infinite which has all the possible perfections. This is an actual and positive sense of infinity while men are considered as finite and imperfect.

The paradox of infinity cannot be rejected as for example the paradox of aether in physics. Even if we ignore the infinity of space or the infinity of atoms, problems such as the finite divisibility or more generally : where we

draw the line of simplicity or of completeness of a whole will always be in our way. Thus we have to turn to our notion of infinity and examine it with respect to limits. The very basic problem we face is whether the notion of infinity is experienced, inferred or posited. In all the cases the infinite appears as within the finite, or, more correctly, through the finite. A definition, a conclusion, a beginning, all are limit - words. Apart from the problem of infinity in the world, the simples of categories when considered from the point of human thought appear as facts made (through statements), formed, assumed, and are consequently finite. The delimitations occur all along the schemata of semantics. Infinity then enters oui consideration as something negative. That which has no limits, that which has no ends is infinite. Against such a position the positive side of infinity is introduced (as in Descartes' philosophy) and is posited. Fundamentally then in terms of limits a first consideration of infinity through the topic negative - positive is possible if we start with a definition of infinity as that which has no limits.

At this point we could advance further through the definition.We could consider the definition as an end of a process (a conclusion) or as a beginning, or as an operative device that would allow us to advance (an arbitrary beginning, or a helpful arbitrary limiting). Thus we would get into consideration of methods and processes.We can do the same thing by abandoning the definition and considering infinity in terms of processes and orders (methods). A set up order involves finitude, but not in the manner that a definition does. Because in a definition one thing is limited by another, is expressed in terms of another, and if we take this hierarchy far enough we will realize that even finitude alone is paradoxical. Things are different in the process.We do not have to find an actual limiting thing always, but we have to center our attention to the process of limiting or of breaking limits. If we set up an order which tends toward infinity (order in all these cases is taken more in the sense of process rather than hierarchy) by a standard process (adding, dividing, limiting, defining etc.) then the infinite does not lie on the actual end of the process but in the process itself, in the activity. This way we can consider infinity not in terms of the actuality of limits but in terms of the potentiality which is in the process of being actualized. This actualization of the potency is what the phrase «tends toward infinity» means.

Potency, actuality and actualization are useful distinctions, but they are by no means the end of the paradox. To this process of actualization we apply the concept of limits: Does the process have any beginning or end? Is the simple operation by which we defined the process, simple and indivisible? Are there limits to this process, and, if there are, are they from the inside or the outside, are they negative or positive? The interplay of the

distinctions positive - negative, actual - potential, limits - order, gives us a spectrum of considerations of the paradox, which spectrum has to be limited, and so we will have to enter an inquiry into the paradox in search for a more complete view.

We made three distinctions in the preceeding passages : infinite - definition, infinite - order, and finally we tried to limit the order by the need to refer it to another limiting whole. These distinctions may be placed in any order of limiting and limited, determining and determinate, etc., but it is all of them which will help us structure our further inquiry. The way we place them (a made order) may be pointing to a way of setting up a system, but it is important to keep in mind that they are flexible and interchangeable, and that they have actually been interchanged in the history of thought. Our point is that since the paradox of infinity challenges the whole of a structure, it will challenge the whole, the parts, the arrangement, and our purpose is to study these relations, as long as we can keep them flexible. One way we can keep this flexibility is by making a matrix, or, which amounts to the same thing, ask questions that will involve all three distinctions both as points of view, and as points viewed. This way we can keep the integrity of the whole safe and the parts flexible, if we keep in mind that the relations between «point of view» and «point viewed» are also flexible in terms of determinacy, indeterminacy, determining, determined, determinate and determinable. Here is the matrix :

Whole	3	3.1	3.2	3.3
Process	2	2.1	2.2	2.3
Definition	1	1.1	1.2	1.3
		1 Fact	2 Thought	3 Things

The problems which arise out of the matrix in relation to the paradox of infinity are of the following characters : Can infinity be predicated of something? (3.1). Can we have an infinite process? (3.2). Is the whole of being infinite or finite necessarily? (3.3). Can we make synthetic a priori judgments? (2.1). What are the limits of the activity of knowledge? (2.2). Can there be a knowledge of the infinite? (2.3).What are the limits of language? (1.1). What are the relations between thought and expression in terms of infinity? (1.2). Can a language, or the words be symbols be infinities of things? (1.3).

In the above set of the questions we can observe that the questions which contain the same numbers but in a different order are alike and they differ

in the direction (point of view - point viewed). The rest which contain a repetition of the numbers (1.1, 2.2, 3.3) are more integrated and their relations will be proved more essential in the final considerations.

In the consideration of these problems the distinctions positive - negative, actual - potential will be useful and furthermore new distinctions will be used such as determinate - indeterminate, definite - indefinite, form - matter, necessary - contingent, absolute - relative, being-nothing (void). These distinctions appear under different orientations in the different problems which center around the finitude or infinitude of things, the limits of the activity of knowledge and the limits of language and expression. Under these problems as points of view we deal with three fundamental problems (instances of the above) : whether we will call anything infinite, whether we can comprehend infinity, and finally whether given the above we can ground infinity necessarily in thought and actuality or whether by positing infinity we can come to the above.

The above problems are structured by ordered schemas of finites, but in the present considerations we are motivated by the paradox of infinity. We implied that the idea of infinity is not experienced, nor imagined. The materials (the «out of which») which we are given (sense and imagination) seem to exclude infinity. But, if we admit that experience and imagination are already structured in terms of finites and orders, then the statement loses its necessary validity. On the other hand we do talk about infinity. These two reasons combined point towards a consideration of the relations of the structured, the unstructured and the structuring activity, and thus they may lead us to a solution of our problem through the examination of infinity in the relations facta - data, made - given, in discourse. This task we will undertake later in the paper. Now, given the second reason (that we do talk about infinity), we can start from there and consider the different ideas (opinions) about the infinite as they are found in the writings of different thinkers. Following the previous distinctions in the formulation of the problems (experienced facts-activity and process-the wholes of things), we will structure the rest of the paper in the same manner. We will first deal with the semantics of infinity, that is, we will take facts made about infinity, and attempt to interpret them, because, if infinity is inferred or posited alone, we, in order to experience it, will have to go to the texts where infinity is inferred or posited. In the second part we will turn to thought and consider the problem in relation to discourse and paradox. In the third part we will finally place the paradox of infinity and its relation to discourse within the whole or some «wholes» of human knowledge, activity, and making, and we will trace relations. Since we are in search for facts (inferred or posited)

we may well begin with a group of thinkers who do actually accept, infer, or posit infinity: Newton, Descartes, Spinoza.

In the amazingly well organized *Principia*, Newton attempts to investigate the laws of the motions of things within the Universe. Out of the three distinctions of our schema of questions he selects the things as a point of view (even in the *Optics*, the observer is left aside by the consideration of corpuscles). Even though he maintains that the motion of the whole is the sum of the motion of the parts Newton does not become a pure atomist, in the sense of stopping at the indivisible (as Lucretius does). He talks of nascent and evanescent quantities and the infinitesimal calculus is an attempt to avoid the paradox of the infinite division in motion. Matter for Newton, composed or not composed out of indivisibles, has a finite quantity. Matter can be reduced to mass, to measurable form, to an actuality (mathematic). By defining then matter as an actual quantity Newton sets up a finite at the beginning. At this point there is no need to speculate about the infinity of the world, even though measurable masses and distances can, as quantities do, increase ad infinitum and there is no need for them not to increase. The argument (process) advances through motion, which is measurable (by virtue of simple composition), and by two sides (through-motion) we reach the form of the Universe mathematically. The infinite then, lies on the Universe. The process may be infinite in terms of eternity (eternity of atoms, incorruptibility of matter-no generation ex nihilo) but it is finite in terms of laws of motion : the process is limited (determined) by the necessity of the laws of motion. The necessity of the laws of motion is guaranteed by an infinite universe. The infinite cannot move (the importance of the discussion of relative and absolute motion) and is one and absolute and therefore necessary. Therefore the infinite as a principle, that is the one, infinite Universe is what Newton posits. But we said that Newton talks in terms of t h i n g s. What he has posited is an infinite in the mathematical actuality, which can merely be an extension of the properties of quantities. He is not interested in a system which works in explaining, but he wants to posit a system which i s. The third book of the *Principia* undertakes to do this task. He has to validate the laws and his method of reaching the laws and thus he begins with the rules of understanding which validate the reduction to the finites (quantities) and the composition of systems out of finites and according to determinate relations. The rules do exactly this with respect to a real, not a mathematical actuality (in the first part geometry is a branch of mechanics). After this we can go on and make the infinite principle an infinite universe. The infinite universe determines the finite, the laws of the process and thus it makes the process infinite but determinate. But since infinity, no matter

if absolute or not, cannot be fully accounted for and consequently the necessity of the determinate laws is very tentative (in other words, why do we not have chance, or how do we know that the mechanistic schema is absolute in the infinity of time and space?) Newton posits a God, governor, infinite, eternal, who, by being infinite, is absolute and determines the course of the universe. Thus by positing an infinity he determines the finites definitely.

Descartes (references: *Discourse* pt. 4, *Meditations* 3 and 4, *Principles of Philosophy* pts. 1, 2, 3.) even less systematic and determinate than Newton, works in a parallel way. Their difference lies on the weight of the selection of the problems. Descartes' point of view is not t h i n g s, but t h o u g h t. His reduction is not a reduction of things or of mathematical objects but of doubt and the clear and distinct ideas of thought are the new finites. They are simple finites of thought. Even infinity, which is a positive idea for Descartes, clear and distinct, is not posited through the things but through thought, and is based fundamentally on the reflexivity of the *cogito ergo sum*, logistically centered on the process of *dubito ergo sum*, or of reduction.

When he turns to infinity in the world, Descartes from the point of view of thought, does not accept it. Matter is potentially divisible ad infinitum, space is potentially extended ad infinitum but both are not actually infinite. They are indefinite (again, the thought selection in the choice of the term).

The infinite is a principle and a positive idea of perfection. It makes the finites determinate, as in Newton, but in a different manner: through thought. So there is a possibility of judgement through the idea of the perfect : The finite is an idea negatively determined since it is derived from the infinite and perfect. On the level of things the indefinite and indeterminate is the negative idea derived from the idea of the finite (limits - that which has no limits). On the level of things the infinite is opposed to nothingness (the logistic void) and matter is identified with extension (a logistic identification from thought selection). On the level of t h o u g h t, the infinite is opposed to the finite. The logistic actualization of matter in extension (from a thought selection) makes extension indeterminate since the void, according to thought, is non existent.

Spinoza (references: *Ethica ordine geometrico demonstrata*, pts. 1,2) is centering his discussion along the lines of thought and his very systematic treatment of the subject allows us to go through the argument (one has to follow definitions, axioms, and propositions in order that he will have argument-process in its clearest form). The absolute infinite in Spinoza's *Ethics* is opposed to the finite or to the infinite which is determined in kind and becomes reflexively substance, the one, only, and absolute substance. Through this infinite, in logistic cause and effect we can understand the

finites. The principle then, even though a limit (the process of thought stops at the infinite) is infinite, and only through the absolute necessity of it we can understand the finite (determined).

The logistic way of thinking then makes finite simples and determines their laws by positing an infinity. This way they can avoid a beginning in change which would make the beginning indeterminate and determinate at the same time, affecting the process accordingly. This way then they avoid the paradox, that is by positing an actual infinite. But, this goes as far as things and thought are selected. Especially, it is the case with the things or when thought centers on the things. Hume will come up with a scepticism which is based again on indefiniteness. When on the other hand we want to keep to the factual level, where the relations of form and matter have disappeared logistically (atom is the s t r u c t u r e of m a t t e r) because it is after the simple fact we are looking, and no cognitive simplicity could arise out of a composite of matter and form, then the paradox of indeterminacy reappears. Because it is not a fact that the world is infinite, or finite, or that the number of the atoms is infinite, or finite. Factually we reach the indivisible, which is indeterminate now, since even the atom is not factual cognitively. But, since there is still in the world an order determined and finite, the principle of indeterminacy has to account for it. The logistics of quanta physics are not as simple as the logistics of Newton, because we need to remain on the level of facts. A primary indivisible (the h/2 of Plank, derived from the facts) is ascertained, not only because we cannot search for other determinants (emotive), but because there are no more determinants (actualization of a finitude with indeterminateness). Chance lies at the roots of necessity and the paradox remains till we try to determine it through the use of probabilities. Heisenberg and Dirac stand on this side of the argument (Heisenberg, references: *Physics and Beyond, Physics and Philosophy*). Probabilities though are essentially indeterminate, and since the facts (which are usually limited to data of experiments) are individual (the repetition of the experiment is not expected to bring similar results), the indeterminacy still remains, pointing to one or the other direction sometimes apparently opposite (wave - particle). From the point of view of facts Heisenberg questions the limits of language and his quest for new concepts (extention of limits) bears witness to it *(Meeting of Athens, 1964)*. [We can get out of the paradox of indeterminacy by the correspondence principle which, though again is an approximation to infinity from a factual point of view].

The indeterminacy in the facts (a paradox related as we saw to the paradox of infinity) was approached in a different way by Niels Bohr, who uses

an operational method and has selected as a point of view not t h i n g s or t h o u g h t s, but f a c t s (N. Bohr: *Essays on Atomic Physics and Human Knowledge* -especially the *Discussion with Einstein on Epistemological Problems in Atomic Physics*- and *Essays 1958-1962*, esp. *Quantum Physics and Philosophy, Causality and Complementarity, The Unity of Human Knowledge, The Solvay Meetings, The Genesis of Quantum Mechanics*). The facts for Niels Bohr are not logistically and cognitively simple but are always related to the observational situation. If we look for simplicity in the facts we cannot but reduce matter and form, or more correctly merge matter and form and in this case merge also the observational situation with the object observed. At this level of finitude (quanta) and because of the merging, we are faced with an individuality in the data and the facta (which are merged since matter and form and observed and observation instrument are merged). As long as we take them as facts, we have to transcend their individuality, and complementarity is one way of doing this. Complementarity is then a determination of the individuals on the level of facts, even though it allows space for indeterminateness, either essentially (because of contradictory results), or because we can always add other determinations (factual ones), and there cannot be a determinate completeness. Factually speaking, we do not have an infinite fact which will determine the finites (no mention of infinite number of atoms or of absolute infinite universe). We do, on the other hand, have a number of complementary facts which, if taken as a whole, does give us a determinacy which will result in explaining the apparent order (the whole of the complementary facts is at any time determined but has posibilities of indefinite increase by addition). The conditions for complementarity do not rest on method, nor do they rest on the things, but on the facts, observational and simple, and both, the determinacy and the indeterminacy of the system rest on the finite individual facts, or, in Schrödinger's terms, in the finite number of questions of facts we can ask to Nature. Given the operational complementarity principle, Bohr does not ask for a new or an extension of the old language, as Heisenberg does, because language is a finite system in which finite, or potentially infinite complementary sides can be expressed. On the other hand he would not disapprove of the Heisenberg proposal for an indefinite extension of the language would not disturb its function, which is to determine the indeterminate by expressing wholes (actually finite, potentially infinite), and in any case language makes this possible by being at the same time determinate and indeterminate.

Kant (references: *First Critique*, the *Prolegomena*) is operational in method but does not rest it on any complementarity system. His selection

of thought instead of facts or things can account for his particular solution to the paradox. He does not merge matter and form, he rather keeps the form as the determining activity which forms the materials given by the senses. (It is important to add here that neither the relation of determinateness, nor the distinction matter - form are in such a simple, one-directional way set. The use of the distinction, especially on the different levels of the transcendental and the metaphysical deduction is variant). The antinomies of fact that we encountered in Bohr are now the antinomies of thought and they involve primarily questions of infinity and finitude. The questions are out of our schema (line of thought): Whether there can be synthetic a priori judgments, what are the limits of the activity of knowing, whether there can be thought of an infinite thing or process. The last question is the antinomical one but, in a whole so interrelated as the Kantian schema is, all the questions are necessarily interrelated closely. Space and time may be infinite or indefinite as forms of intuition, but in terms of the world and the whole of experience, both assertions, the one of the limited and the one of the unlimited are in an antinomical way related to the whole of reason. In such a situation there can be no complementarity of facts to solve the problem (a thing that goes to prove that the criticism of Kant either by quanta physicists with respect to causality and determinateness, or by philosophers who keep to the factual level, is bount to be in vain, as long as people keep to their own level of selection, without realizing another for Kant). From the selection of thought as a frame of reference, the antinomies signify the boundary of the activity of Knowing. It is not a limit from the inside but a boundary from the inside. It is a boundary which is set by the mind, because of its activity, and is a limit to the mind and its activity (mind and activity are the same as is revealed in the act of judging). In the *First Critique* (Phaenomena and Nooumena) and in the section on Hume and scepticism in the *Discipline of Pure Reason*, Kant sets the limits of knowing. In both cases Hume can be taken as the start but the scepticism of Hume uses indefiniteness to attack both the finite and the infinite and ends in scepticism. According to the famous metaphor of Kant (*Geography and Reason*), Hume thinks of reason as of an indefinitely extended plane, while Kant says that it is more likely as a sphere (limited by its own properties). In this sphere, the three questions are closely interrelated. The possibility of synthetic a priori judgments resting on the activity of the formal categories (categories are functions of unity in judgments) and on experience (sensible manifold) make up the content of the sphere of possible knowledge while the problematic nooumena (thinking is not knowing) make the boundaries of the sphere; they are problematic (they are determined as problematic) by the activity of knowing itselt, in this

way they are boundaries from the inside. In such a schema even the beginning principles (the principle of Apperception) are not absolutely beginning (and thus limiting, determining, finitizing) points, but rest reflexively on the whole of the sphere, the whole of the self-centered activity. The principle is like the appex of a cone which is an appex because of the rest of the cone and the rest of the cone is such because of the appex which determines it. This whole (sphere) is limited from the inside by self imposed limiting devices which make it a whole. Questions about the Absolute unlimited, «*das All*» (Spinoza) are rejected by the category of limitation (limitation is combination of reality and negation) and it is through the employment of categories that we have synthetic a priori judgments.

Einstein (references: *Principle of Relativity, Evolution of Physics, Meaning of Relativity, Essays in Physics, Can Quantum-Mechanical Description of Physical Reality be considered complete?)* is opposite to the uncertainty and complementarity of Heisenberg and Bohr and his attempt is to set up and explain the order in the world. As selections out of our schema he takes the things. Einstein's selection is a strongly metaphysical one, much more strongly so than Aristotle's in his *Physics*. Placed thus between, Einstein could be studied with reference to Bohr and Heisenberg on the one hand, and Aristotle on the other, in order that we get a clearer view of his system.

There is a usual basic misconception of Einstein as a «relativist». It is true that in the special theory of relativity and in his discussion with Bohr and Heisenberg, Einstein establishes observational physics, where the frame of observation is inseperable from physical reality (relative space and time). It is also true that he conducts in there operational thought experiments (favorably recorded by the two opponents). But we believe that the special relativity theory (as well as the classical physics or the field physics) is for Einstein only a dialectical step towards the theory of general relativity which rests on the selection of things rather than observational facts. This is not to say that the theory of special relativity is wrong as in a dialogue. If we consider that it is physics we are dealing with, and also the particularities of the dialectical method, we can see that classical physics, the special theory of relativity, Maxwell's equations are true, in the sense of application to specific cases but the general theory of relativity which assimilates all these and comes with a theory for all the Universe is what Einstein center his search around. On the other hand, complementarity is not a dialectical activity even though it assimilates opposites, because it does not assimilate in terms of a whole higher truth, but restricts the truth to the different sides of the debate on facts. Furthermore Einstein does not center his discussion

in terms of finite, discontinuous quanta, but rather around a space - time continuum which finally becomes an actualized geometrical finite. In terms of physical finites Einstein uses the finite speed of light not a finite of indeterminacy like the constant of Plank.

In relation to Aristotle (same selection: things), Einstein agrees on the continuum of space and time. The difference between «energeia» of Einstein and «entelecheia» of Aristotle's *Physics* shows clearly, that as far as physics go, Einstein is more metaphysical. To expand this further : starting from the problem of gravity Einstein advances as far as to reach a geometric universe. In it, the physical reality is the field, that is to say, energy, actualization, actuality. A further dialectical attempt was made to reach the unified field theory of pure actuality but it remains open and Einstein accepts the double reality of matter and field as a basis for further dialectical search towards the pure actuality. Aristotle does not reach the energeia, the pure actuality before the *Metaphysics*. In the *Physics* the prime mover is a moving cause and not the final cause of the book Λ of *Metaphysics*. The beginnings are set from one direction, but indefiniteness lies in «the after». Necessity sides with the potentiality of matter and the power of the unmoved mover and is the necessity of «the before», the necessity of the process. Einstein in his general theory of relativity, aims at his principles, works towards his principles, which is a fully actualized field (a principle of art rather than of nature-depending of course on the distinctions that one makes between art and nature). The form-determining «art» principles are more basic rather than the moving principles. Whether coming from a supreme being or not (the divine artificer, who, by the way, does not play dice), determinateness is expressed in the actual form of the unified field that the universe is. Under such a schema the Universe in finite. It is not a universe of motion, as is the physical universe of Aristotle, since it is a whole which involves time as one of its vectors. The universe is finite but unbound. In this sense it resembles the sphere, and, distantly, the Aristotelian universe. The finitude of the universe is based on the density of matter (definite) and the speed of light (finite).

About Aristotle, we said something about beginnings, ends, determination, motion, continuity (all words relating to our paradox). What is relevent now is to determine the place of the infinite in his thought. He starts an inquiry into Nature which is rendered as an internal principle of motion. The definition of motion which rests upon the process and relates the actual and the potential is a beginning in the discussion of the infinite. For, out of this definition, we can move to problems of the continuity or eternality of motion, to continuity and infinity of matter, of time, place, numbers, magnitudes etc.

All these questions require the notion of infinity to be solved and thus Aristotle has to define infinity before he proceeds. From the standpoint of things infinity cannot be attributed to facts. The continuity of matter does not allow for a finite, actualized atom. There cannot be an infinite body or an infinite magnitude. The numbers may allow for a formal infinity, but this is mathematical and not physical. There is though an infinity in the process, not in the fact (or the attribute of a thing). The process of adding or dividing is progressing ad infinitum. Aristotle does not define (limit) the infinite as that which has no limits, but places it in the realm of potentialities. It is a potentiality which is never actualized, because there is always something further, and as long as there is something further the process is infinite. Together with the notion of the progress we have the notion of the motion and the notion of time. Motion is continuous and eternal and so is time. Matter remains always a potentiality, parallel to the one of infinity, and a necessity (as we saw infinity to be in other thinkers). Basic in all these discussion is the notion of time, eternal, continuous, determinable in terms of indivisible «nows». The process even though it establishes the infinite as a potential being, is not itself unquestioned in terms of determinateness. Is it infinite, or does it have a beginning? Is it possible to have infinite series of movers and moved, who are actualized through the process of motion at the same time, or does it progress out of one mover, one actuality? The infinite series of movers and moved actualized at the same time (as in Descartes) would create a paradox of an infinite actuality within a finite reality. We would be led then to a paradox similar to Zeno's, where infinity is placed within finitude as an actuality. The finitude of the series together with the eternality of motion and time call for a moving principle (limit) which is the beginning of motion but also the seat of the infinity of motion. A reflexivity then between the finite and the infinite sets up the beginnings and the limits for intelligible and perceived motion. The paradox is placed at the limits of reality and intelligibility and it is not only the finite but also the infinite which is involved in the limits. The universe then as an actuality within finite times («nows») is finite, because space cannot be infinite (no infinite actuality) but the same universe is unbound in terms of potentiality of motion and eternality of time. The circular motion (roughly parallel to the hypersphere of relativity) answers these basic requirements. On these boundaries of the knowledge and reality of things (natures) the paradoxical relations of infinity - finitude are placed. An unmoved mover who determines by being unmoved the indeterminate (the eternal motion). At this point the relation is clearly reflexive. A further consideration of infinity lies at the infinite power of the unmoved mover. The unmoved mover is not like time

infinite potentiality and finite actuality, but is indivisible (no magnitude or actuality in the terms of sense experience) and actual but has infinite potentiality (power) to cause motion.

In the *Metaphysics* the unmoved mover as a final cause accounts for the unity and order of the universe (κόσμος) through the pure actuality (no matter) of reflexive, self organizing and organized thought (principle). At this point we can find no relations to the infinite for we have moved towards pure actuality, pure form which determines the order of the kosmos not only in terms of the antecedent as in the *Physics*.

Out of the semantic examination of these thinkers (based strictly on the matrix and the problems we made up out of it) there came out the paradox of infinity - finite, determinate - indeterminate, definite - indefinite, structured - unstructured as a boundary condition of activities, beginnings and ends of things, determinacies and indeterminacies of facts. It is also clear in these orders of hierarchies or processes the matter (potential) and form (actual) distinction (a distinction merged, reversed, emphasized, used) is a problem. The problem, then, of infinity was always solved in terms of complete separation of matter and form. One way leads to matter unformed, purely potential, the other to form purely actual with no matter.

The God of Newton and Descartes, the prime mover of the *Metaphysics*, the infinity of Cantor and Hilbert, the probabilities of the Quanta, the unified field of relativity, the autonomy of Kant's Ethics are all actualities which answer the paradox. The purely posited actuality (finite or infinite) accounts in a pure formal fashion for the definiteness of the orders of finites and processes.

On the other hand, the discussion of the infinite seeks an end in the consideration of matter. Matter as a potency is close to infinity. The infinite in the process is one instance of this. The infinite power of the prime mover of the *Physics*, the «things-in-themselves» (and also the relation of objects given and objects thought at the beginning of *Transcendental Logic*) of Kant are another attempt to deal with the problem of infinity. Even when we merge matter and form in the logistic structure of matter, or in the factual level of the simples, the infinity is still there as a probability, a possibility, a potentiality. But as we define, and order, and finitize, and form, any talk on the level of matter does not satisfy us. Thus we go to the *Metaphysics* to account for the necessity of the *Physics* in a formal fashion, or in Kant we encounter material boundaries (nooumena) but we do not stop here: either through the formalities of autonomous action, or through a future metaphysics we go further in the relation of formal and actual. The God of

Newton, Descartes, and Spinoza as infinite and actual takes the probability out of the material and establishes determinacy.

What is fundamental in here is the paradox as an interplay of form and matter. The paradox accounts for things - thoughts - facts and is on the one hand a motive force (matter and necessity) and a limiting force (form and necessity). And it is so in the whole of knowledge. It seems then that our broad presupposition that the human thought works in terms of finites and orders is not complete since it operates, advances, is organized and made into a whole through fundamental paradoxes, such as the paradox of infinity. Let us critically study the paradox which motivated and structured our paper.

In the preceding pages we structured a discussion of the finite in terms of the finite facts, thoughts, things constructed and in terms of selections of view points. Flexibility then arose out of the various combinations of these factors. We were able to some extent to determine the indeterminate but through finites. And it is in this that there is still paradox. If we want to carry our inquiry further we would have to critically question the construction of our schemas, and its finite questions.

In both dimensions the schema is made out of finites but it is made possible by delimiting an area of flexible relations between the point of view and the point viewed. To put it in our terms, we made a distinction between selections and the rest (facts, thoughts, things) which are all made. The finite schema was made possible because even the vertical dimensions (selections) were structured in terms of the same finites (definition, process, whole). In a certain sense then we left the paradox unanswered. The «point of view» and the «point viewed» delimit an area of flexible interrelations of indeterminacy but also determination (relative). What is the place of the infinite in it? The nine questions either expressed a reflexivity in their interrelations, or a «wholeness» and integrity (1. 1, 2. 2, 3. 3). In the interrelations of these three we could find some clue for our problem. But most of all a second consideration should be added so that we will try to escape from the made, the formed and finite. This we will take out of the nature of selections, not out of the made selections, but out of the obscured unqualified data, out of the «given» (to the extent that we can). It could be the case that we are still talking in terms of the matter (given) - form (made). We can accept this with the presupposition that these relations can interchange or be related flexibly. We still need to criticize our schema even if it is impossible to get out of our finites and our forms.

If we take our selections and our systems, not as points of view - points viewed but as data qua data and facta qua facta respectively, and if we also take our paradox (infinite - finite, determinate - indeterminate) there are

then four possibilities determined: 1. There are infinite given things and infinite things we can make out of them (infinite data-infinite facta). 2. There are infinite given things and finite things we can do with them (infinite data - finite facta). 3. There are finite things in the world and finite things we can do with them (both finite) and 4. There are finite things in the world and infinite things we can do with them. In the above determinations things are to be taken as in something or as merely the given. We can also, in the above schema, replace finite - infinite with determinate - indeterminate, but this does not mean that they will side in the same way that the finite - infinite sides, because the same interrelations can be established between the finite-infinite and the new distinction (the finite is determinate or indeterminate). We can also reverse the direction of the four determinations, start from facta and go to data. We can also see that these distinctions do not remain in the abstract but go into our schema (in facts, things, thoughts). To illustrate the above: It may be the case that the given is structured and thus it structures our thoughts or the reverse, or the opposite, or the reverse and the opposite combined. Another example out of the concretes I take it out of the things: It may be the case that the whole structures the part, or the reverse, or the opposite, or the reverse and the opposite combined. The same may happen with universals and particulars, real and apparent facts, symbols and objects. It can also be the case that data and facta are not distinguishable (dogmatism) but are one and the same, in which case we undercut our project but still we can take the case as a determination out of the indeterminate (flexible relations) but determinable. Or it can be the case that facta and data are completely separate, in which case it is meaningless to talk about their relations (scepticism). Again this is a determination which determines and destroys the flexibility. These last two examples ask for a more critical insight to our problem.

Data are taken as individuals. The word individual itself means indivisible, atomon, a finite simple. Two problems then arise out of this. The problem of analyzing data and the problem of communicating data (from the facta point of view). If a datum is individual, indivisible, absolutely simple and single, then how can we analyze it? Also, how can we communicate it? We come then from the point of view of facta and try to analyze the datum and go beyond the indivisible by division; or we extend the individual by communicating it, by adding, by putting it in a whole. Communication and analysis run parallel, for we may analyze something by dividing, or by placing it in a larger whole (communication in terms of data). Communication is in terms of a whole (other people, a language etc.) but may be achieved by analysis since we use individual words with general meanings, and when

in doubt we define, or analyze our discourse in terms of significant terms in it. To illustrate this in an example related to our paradox: Data are taken as infinite while what we construct is finite. But this is not absolutely the case for indefinitely many scientists may have a different idea about a phenomenon. To solve the problem they have to turn to the data as finite and thus as determining. In the same line of thought ambiguity is a fact referring to many kinds of data. Meaning and reference then fit the schema of consideration of our paradox and can be explored through it.

The above discussion focused on the paradox. Whether there is in fact (or in texts) a paradox. Whether we can conceive it as such. And finally wherefrom it arises. The discussion has a parallel theme running in the opposite direction: Whether we make paradoxical facts. Whether knowledge is paradoxical. Whether it is the paradox which generates and forms knowledge. The paradox then was the motive force and the end, it is a material necessity and a forming (a limiting and ordering) force for knowledge. It sets us going and it also limits us. It appears that we are at one and the same time motivated and restricted by the paradox. At the boundaries of knowledge then we do not stop blocked by the outside. The flexible relations we set up tell us that what is determined and what is undetermined close up an area but also have the potential for further expansion: they are at the same time boundaries and forces from the inside. Now we can see the relations within the paradox, in a less negative way, not as the determinate and the indeterminate but as the determinable and the determining, the matter and the act. Of course the matter and the act can interchange in being attributed to the determinable or the determining, or merge, or enter in whatever flexible manner, because the matter may have some form, and determining as a process may be a potency. And furthermore, the activity and the potentiality, set up in flexible paradoxical relations, are related with other paradoxes at this level of limits. To give an example: from the activity side of the relation we can be tied down to the paradox of freedom, from the potentiality side of the relation we can be related to necessity. If we take into account the relations freedom - necessity we can get a sense of the importance of the paradoxes in Human Knowledge. On the other hand we connect things in another way: potentiality with freedom and form with necessity.

In the previous passage I purposely increased the range of the paradox by relating it to other paradoxes such as to making up form and to freedom and determinism. On the one hand we could make our case stronger for the philosophical importance of the paradox, on the other hand we could make a start on the consideration of the paradox outside of the theoretical sciences.

Out of our schema we can draw parallel relations in terms of the whole of human knowledge, activity, and making. The middle line of questions (1.1, 2.2, 3.3) show us possibilities for parallel relations: to facts as facts though as activity, and things as Being we can parallelize History - Poetry - Philosophy, or Physics - Epistemology - Metaphysics, or Theoretic - Practical - Poetic. Now, if it is through an art of semantics that one wants to approach the subject, one would have only to take the specific references and see them under the light of the questions that we set up, and perhaps modify the questions in terms of the new paradoxes. If on the other hand, it is an art of discourse and an inquiry that we are aiming at one must go beyond the matrix to the data - facta relations, with respect to the new paradoxes.

The problem of infinite - finite, definite - indefinite ranges through all the sciences. In the poetic sciences, which may primarily appear as opposed to the infinite, since they are commonly seen as ways of form - giving, the paradox of limits is of no less importance. For what is it which determines the form? Is it the poet? If it is, then the problem of knowledge reappears. If the poet is describing things with magnitude and limits, is the determinateness of the magnitudes of the poema a result of the determinateness of things or of the poet? Is the immitation of reality a copy of the world or of the poet and in the first case how do we explain the different versions of a thing in art, or if it is the second, what are the relations of the poet to the material he uses in his different poems? The same problems appear at the criticism of art. And, if art is an expression of something is the language or the clay limiting or adding to the expression? If that which is to be expressed is infinite then how can we express it in finite forms? Is there any truth in a statement of a poet that the best poetry is the unwritten one? Discussions about art then would center around the different magnitudes of the poema, around its unity and its order, around problems of expression, imitation and limitations or potentialities of art in the forms or in the materials, which can range from the whole universe down to the instruments of utility.

In history, in the interplay between the actual events and the writing of history (ordering and limiting) we can find the paradox. What is that determines what in history? Problems of complete or incomplete induction, problems of detailed analysis of causes, and problems of construction, of writing of history are all related to the paradox of infinite - finite, determinate-indeterminate. We thus have a range of histories from the simple tape-recording and the camera to the chronicles to the quantitative histories, to history as it should be. Another paradox enters history, that of freedom and necessity, which borders with the infinite and brings us closer to another area, that of action and the practical.

In the sphere of action problems-paradoxes arise out of consideration of infinity. Is the action a principle, a way of getting out of limits? Does it involve the infinite potentialities of matter or is it restricted by the forms of behaviour or other forms? Is action aiming at freedom or does it require freedom? Is action autonomous with respect to forms? Does its determination involve knowledge of the determinate or indeterminate in the world? These questions were answered antinomically or paradoxically by different thinkers and people, and there is no definite answer to them. Is it not the case that we are again faced with the dynamical paradoxes which limit from the inside our knowledge and our world?

The paradox of infinity, limits and order was the motive force in the writing of this paper. It also structured the paper and finally gave limits to it in a flexible manner from the inside. It is then partly by the limits that the paradox established and partly by the finitude of time that we will have to give an end to the study of paradoxes and of infinities.

ΑΠΕΙΡΟΝ, ΟΡΙΑ ΚΑΙ ΤΑΞΙΣ

ΜΙΑ ΜΕΛΕΤΗ ΓΙΑ ΤΟ ΠΑΡΑΔΟΞΟ

Π ε ρ ί λ η ψ η.

Ἂν δεχθοῦμε ὡς βασικὴ προϋπόθεση ὅτι ἡ ἀνθρώπινη σκέψη κινεῖται μὲ βάση τοὺς ὁρισμοὺς καὶ τὶς τάξεις, ὁρίζοντας δηλ. ἔννοιες καὶ βασίζοντάς τις σὲ σειρὲς καὶ τάξεις αἰτιότητος, τότε καταλαβαίνομε γιατὶ τὸ ἄπειρο παρουσιάζει ἀπροσπέλαστες δυσκολίες στὴν ἀνθρώπινη σκέψη. Σύμφωνα μὲ τὴ φύση του τό ἄπειρο δὲν μπορεῖ νὰ ὁρισθῆ, νὰ περιορισθῆ, καὶ συγχρόνως δὲν μπορεῖ νὰ τοποθετηθῆ σὲ σειρὰ μαζὶ μὲ ἄλλες αἰτίες καὶ ἀποτελέσματα, γιατὶ τότε οἱ σειρὲς καὶ οἱ τάξεις ἐκτείνονται ἀτελεύτητα καὶ γίνονται κενὲς καὶ ἄχρηστες, χωρὶς ἀρχὴ καὶ χωρὶς τέλος. Δύο προσπάθειες ἔχουν γίνει γιὰ νὰ λυθῆ τὸ πρόβλημα αὐτό, ποὺ βρίσκεται στὰ ὅρια τῆς ἀνθρώπινης σκέψης. Ἡ πρώτη εἶναι νὰ ὁρισθῆ μαθηματικὰ τὸ ἄπειρο, χωρὶς νὰ ὑπάρχη ἀνάγκη νὰ τὸ παραθέσωμε σὲ ἀντιστοιχία μὲ τὸν κόσμο: τὸ μαθηματικὸ (ἐνεργείᾳ) ἄπειρο ὑπάρχει ὡς αἴτημα, ἔτσι ὥστε νὰ μὴ εἶναι κἂν ἄπειρο οὐσιαστικά. Ἡ δεύτερη προσπάθεια εἶναι νὰ τοποθετηθῆ τό ἄπειρο στὸν χῶρο τῶν ἀκατανοήτων μυστικῶν ὅρων τῆς θρησκείας, οἱασδήποτε θρησκείας. Σ' αὐτὴν τὴν περίπτωση τὸ μόνο ποὺ κάνομε εἶναι νὰ

ἐπαναλάβωμε ὅτι τὸ ἄπειρο εἶναι ἀκατανόητο καὶ νὰ ἀξιώσωμε ὅτι ὑπάρχει.

Παρ' ὅλες τὶς δυσκολίες παρατηροῦμε ὡστόσο ὅτι οἱ φιλόσοφοι προσπαθοῦν νὰ ὁρίσουν τὸ ἄπειρο. Ἂς ἀντιμετωπίσωμε λοιπὸν καὶ ἐμεῖς τὸ πρόβλημα. Ἀρχίζομε μὲ μιὰ γενικὴ θεώρηση τοῦ προβλήματος, ποὺ θὰ δώση μορφὴ στὴ συνέχεια τῆς ἔρευνας. Εἶναι τὸ ἄπειρο ἰδέα ποὺ ἀνήκει στὴν ἐμπειρία ἢ εἶναι συμπέρασμα σειρᾶς συλλογισμῶν ἢ μήπως εἶναι μιὰ ἀρχή, μιὰ θέση, ποὺ τὴν δεχόμαστε ὡς αἴτημα ἢ ἀξίωμα; Παίρνοντας αὐτὲς τὶς τρεῖς πιθανὲς θέσεις καὶ συνδέοντάς τις μεταξύ τους μποροῦμε νὰ δημιουργήσωμε ἕνα σχῆμα μὲ ἐρωτήσεις πάνω στὸ ἄπειρο, ποὺ μποροῦν νὰ μᾶς χρησιμεύσουν στὴ μελέτη τοῦ ἀπείρου ὡς μέρους τῆς ἐμπειρίας. Ὅλοι συμφωνοῦν, πὼς τὸ μόνο ἀναμφισβήτητο μέρος τῆς ἐμπειρίας, ὅπου ἐμφανίζεται τὸ ἄπειρο, εἶναι τὰ ἔργα τῶν φιλοσόφων, ὅπως τοῦ Νεύτωνος, τοῦ Καρτεσίου, τοῦ Σπινόζα, τοῦ Χάϊζενμπεργκ, τοῦ Ντιράκ, τοῦ Νήλς Μπόρ, τοῦ Κάντ, τοῦ Ἀϊνστάϊν καὶ τοῦ Ἀριστοτέλους. Τὸ σχῆμα τῶν ἐρωτήσεων ὅμως, ποὺ χρειάζεται γιὰ τὴν ἐξέταση τῶν φιλοσόφων αὐτῶν, βασίζεται πάνω σὲ ὁρισμοὺς καὶ τάξεις, καὶ γι' αὐτὸν τὸν λόγο ὑπόκειται σὲ κριτικὴ ὡς πρὸς τὸ πόσο μπορεῖ νὰ μελετήση τὸ ἄπειρο.

Συμπερασματικὰ τὸ πρόβλημα τοῦ ἀπείρου μπορεῖ νὰ ἐξετασθῆ στὸ τελικὸ ἐπίπεδο τῆς γνώσης ποὺ δημιουργεῖται ἀπὸ τὴ σχέση δεδομένων καὶ γεγονότων (τῶν data καὶ facta) τῶν στοιχείων ποὺ μᾶς δίνονται καὶ τῶν πραγμάτων ποὺ κάνομε, ποὺ καταλαβαίνομε καὶ ποὺ κατασκευάζομε μὲ αὐτὰ ποὺ μᾶς δίνονται. Στὶς ἀκόλουθες τέσσερεις λογικῶς δυνατὲς θέσεις ἐντάσσονται ὅλοι οἱ φιλόσοφοι τοῦ παρελθόντος καὶ ὅλες οἱ πιθανὲς ἀπόψεις γιὰ τὸ ἄπειρο: 1. Τὰ δεδομένα εἶναι ἄπειρα, καὶ ἄπειρα εἶναι αὐτὰ ποὺ ἐμεῖς μποροῦμε νὰ κάνωμε (οἱ θεωρίες) μὲ βάση τὰ δεδομένα. 2. Τὰ δεδομένα εἶναι ἄπειρα, ἀλλὰ ἡ ἀνθρώπινη σκέψη μπορεῖ νὰ κάνη μόνο περατὰ πράγματα. 3. Τὰ δεδομένα δὲν εἶναι ἄπειρα· ἄπειρα πράγματα (θεωρίες κλπ.) μποροῦν νὰ γίνουν μὲ αὐτά. 4. Τὰ δεδομένα δὲν εἶναι ἄπειρα, ὅπως δὲν εἶναι ἄπειρα καὶ τὰ πράγματα ποὺ μποροῦν νὰ γίνουν μὲ αὐτά. Οἱ τέσσερεις αὐτὲς θέσεις ἐξαντλοῦν τὶς σχέσεις μεταξὺ τοῦ τί μᾶς δίνεται καὶ τοῦ πῶς ἐμεῖς τὸ κατανοοῦμε.

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ΚΥΡΙΩΤΕΡΑΙ ΣΥΝΤΟΜΟΓΡΑΦΙΑΙ ΤΟΥ ΤΟΜΟΥ 3 (1973)

AGPh	Archiv für Geschichte der Philosophie
ΑΦΘΕ	Ἀρχεῖον Φιλοσοφίας καὶ Θεωρίας τῶν Ἐπιστημῶν
CAG	Commentaria in Aristotelem Graeca
CQ	Classical Quarterly
DG	Doxographi Graeci
ΕΕΦΣΠΑ	Ἐπιστ. Ἐπετηρὶς τῆς Φιλοσ. Σχολῆς τοῦ Πανεπ. Ἀθηνῶν
JHS	Journal of Hellenic Studies
JPh	Journal of Philosophy
ΠΑΑ	Πρακτικὰ Ἀκαδημίας Ἀθηνῶν
PACA	Proceedings of the African Classical Association
PG	Patrologia Graeca
P.U.F.	Presses Universitaires de France
RE	Real - Encyclopädie
REG	Revue des Études Grecques
RhMus	Rheinisches Museum
RPhilos	Revue Philosophique
RPhL	Revue Philosophique de Louvain
RSF	Rivista Critica di Storia della Filosofia
SVF	Stoicorum Veterum Fragmenta
VS	Die Fragmente der Vorsokratiker